



# **Innovative Sudoku Designs with Rectangular Sub-zones: Enhancing Experimental Precision in Paddy Field Trials**

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## **Authors' contributions**

*This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.*

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## ABSTRACT

This study investigates the application of a Sudoku square design with rectangular subzones for a field experiment on paddy cultivation conducted in 2022-2023 in West Bengal, India. Traditional row column experimental designs, such as the Latin Square Design (LSD), have been widely used in agricultural research. However, these designs have limitations in estimating a variety of effects due to their structure. The research addresses this limitation by employing a sudoku square design, which has introduced a notable framework for experimental analysis. Four distinct statistical models were employed to analyze the yield data. The models captured varying sources of variation. Type 1 identified three: row block, column block, and subzone. Type 2 added rows within BR and columns within BC. Type 3 included H-square within RB and V-square within CB. Type 4 combined all, revealing six sources in total. Although not all additional sources were statistically significant, variations such as treatment, row, subzone, row block, H-square within RB, and V-square within CB showed statistical significance. Model demonstrated the enhancement of their analytical capability to estimate the effects of these additional sources. The varying number of additional sources identified by each model highlighted the flexibility and robustness of the Sudoku square design in capturing complex effects in the experiment.

**Keywords:** *Latin square design; row column design; rectangular subzone; sudoku square design.*

## 1. INTRODUCTION

Sudoku, a globally popular Japanese mathematical puzzle, originally named “Number place”, is a fascinating example of a specialized case of experimental design known as Sudoku design. In this type of design, an  $n \times n$  grid is sub divided into  $n$  (any positive integer) regions, each containing from number 1-  $n$ , each of the number must appears exactly once in every row, column, and sub zones of the grid. The structured layout of sudoku grid reflects the principle of statistical designs used in agricultural experiments (Sarkar & Sinha, 2015). Baily et. al. (2008) made a valuable addition to the study of sudoku square designs as Gerechte designs by specifically investigating the exclusion properties and exploring the impact of variables such as sub-zones. Which provides more details understanding of experimental results. Similarly, Subramani & Ponnuswamy, (2009) and Subramani et. al (2012) discover that in an agricultural experiment, Sudoku designs are beneficial for studying multiple factors including outcomes like seed varieties, soil fertility, water quality, cultivation methods and types of fertilizers. Fontana (2011), discussed the mathematical underpinnings of Sudoku-based designs, illustrating an application to complex experiments. An experiment involving higher-order interaction or sometimes multiple nuisance factors may be solved by Sudoku, which could be instrumental in improving the statistical efficiency of field trails (Saba and Sinha, 2014). Shehu and Danbaba (2018) studied variance components for Sudoku Square design models

using ANOVA, estimated variance components for developed models, and provided significance tests for various effects. Subramani (2018) highlighted different variants of Sudoku square designs and optimizing criteria for future projects, which enhance their utility and application across diverse disciplines. In most cases, the fixed symmetrical combinations cannot be accommodated in agricultural experiments, asymmetrical or rectangular sub-zones offer better statistical significance and is flexible for accommodating irregularities in the shapes of fields and varying sizes of land. Dauran *et al.* (2020) introduces balanced incomplete Sudoku squares (BISS) design, proposes its construction using orthogonal Sudoku squares, and modifies models to incorporate BISS design. Shehu *et al.* (2023) discover an estimator for missing values in a Sudoku square design without losing important information or deleting rows/columns. A numerical example is provided. Sudoku squares with rectangular subzones also facilitate improved randomization and replication strategies, ensuring treatments are evenly distributed and replicated in a balanced manner across the experiment (Hui-Dong and Ru-Gen, 2008, Saba and Sinha, 2014, Bailey et al., 2008, Bufler et al., 2007, Subramani and Ponnuswamy, 2009).

Keeping in mind, the importance of Sudoku square designs with rectangular sub-zones, as mentioned above, a field-level experiment was conducted in West Bengal on a paddy field where treatment as Seed variety, Row block as different irrigation methods, Column block as

tillage operation, Rows as fertilizer application, Columns as pesticides and Sub zones as different weeding methods can be applied for different variant of the Sudoku square designs to ensure that research efforts should have meaningful insights.

## 2. MATERIALS AND METHODS

### 2.1 Experimental Details

The experiment was conducted at Jaguli Instruction farm, which serves as a university research farm for Bidhan Chandra Krishi Vishwavidyalaya, West Bengal, India. (22°93' and 83°53' E). The experiment was conducted during the rainy season (July-August 2022) with winter harvesting, following the traditional 'Aman paddy' approach practiced in West Bengal. Primary data was collected from experimental plots during the 2022-2023 crop season.

### 2.2 Treatments (Variety) Details

Six varieties of paddy were selected for investigation namely, Gontra Bidhan-1(1), Gontra Bidhan-3 (2), Kalyani-2 (3), Nayan Moni (4), Bidhan Suruchi (5), and Lal Swarna (6). All the chosen varieties are all non-basmati, long grain types known for their excellent yield potential and relatively short maturation periods. In the current study, we used a sudoku square layout with six sub-zones of order 3 x 2.

## 3. METHODOLOGY

The methodology for analyzing Sudoku square design is divided into four distinct types of ANOVA models. The analysis procedure for each model is presented sequentially.

### 3.1 Sudoku Design with Type- 1 Model

$$Y_{ij(klpr)} = \mu + \alpha_l + \beta_p + \tau_k + r_i + c_j + s_q + e_{ij(klpq)}, \quad (3.1)$$

where,

$Y_{ij(klpq)}$  = Response of  $k^{th}$  treatment effect in  $i^{th}$  row,  $j^{th}$  column and  $q^{th}$  subzone effects with  $l^{th}$  row block and  $p^{th}$  column block effects.

$\mu$  = General mean effect

$\alpha_l$  =  $l^{th}$  Row block effect

$\beta_p$  =  $p^{th}$  Column block effect

$\tau_k$  =  $k^{th}$  Treatment effect

$r_i$  =  $i^{th}$  Row effect

$c_j$  =  $j^{th}$  Column effect

$s_q$  =  $q^{th}$  Sub zone effect

$e_{ij(klpq)}$  = Error with mean zero and variance  $\sigma^2$

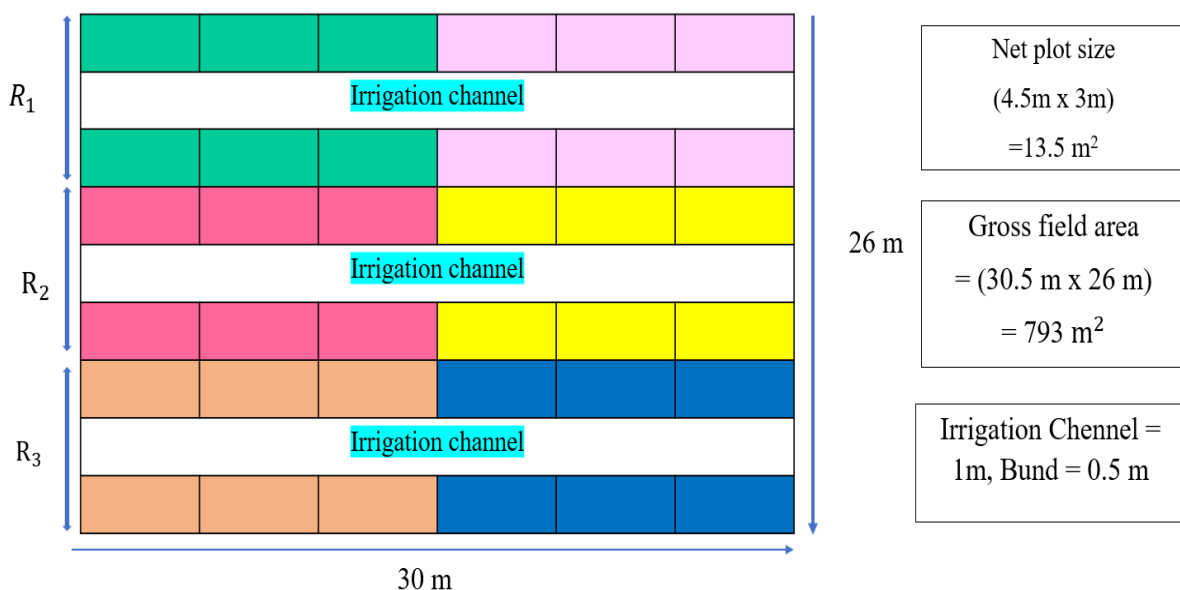


Fig. 1. Layout for the experiment

In addition to estimating row, column, and treatment effects, the above model also allows the estimation of three additional effects: Row block, column block, and subzone effects.

The order of the Sudoku design is represented as  $m \times n$ . After applying some algebraic transformations, we derived formulas for the various sums of squares and their corresponding degrees of freedom.

$$\text{Grand total, } G = \sum_i^{mn} \sum_j^{mn} Y_{ij},$$

$$\text{Correction factor, } CF = \frac{GT^2}{N}, N = (mn)^2,$$

$$\text{Total sum of square, } TSS = \sum_i^{mn} \sum_j^{mn} Y_{ij}^2 - CF$$

$$\text{Tr. SS} = \sum_{k=1}^{mn} \frac{Tr_k^2}{mn} - CF; \quad Tr_k^2 = \text{Sum square of } k^{\text{th}} \text{ treatment total, } k = 1, 2, \dots, mn$$

$$\text{RBSS} = \sum_{l=1}^m \frac{RB_l^2}{m n^2} - CF; \quad RB_l^2 = \text{sum square of } l^{\text{th}} \text{ Row block, } l = 1, 2, \dots, m$$

$$\text{CBSS} = \sum_{p=1}^n \frac{CB_p^2}{n m^2} - CF; \quad CB_p^2 = \text{sum square of } p^{\text{th}} \text{ Column block, } p = 1, 2, \dots, n$$

$$\text{RSS} = \sum_{i=1}^{mn} \frac{R_i^2}{mn} - CF; \quad R_i^2 = \text{Sum square of } i^{\text{th}} \text{ row total, } i = 1, 2, \dots, mn$$

$$\text{CSS} = \sum_{j=1}^{mn} \frac{C_j^2}{mn} - CF; \quad C_j^2 = \text{Sum square of } j^{\text{th}} \text{ column total, } j = 1, 2, \dots, mn$$

$$\text{SZSS} = \sum_{q=1}^{mn} \frac{S_q^2}{mn} - CF; \quad S_q^2 = \text{Sum square of } q^{\text{th}} \text{ Sub zone total, } q = 1, 2, \dots, mn$$

$$\text{Er. SS} = TSS - (\text{Tr. SS} + \text{RBSS} + \text{CBSS} + \text{RSS} + \text{CSS} + \text{SZSS})$$

**Table 1. ANOVA table for Sudoku design with Type- 1 model**

Source of variance	Sum of Squares (SS)	Degree of freedom (df.)	Mean sum of square (MSS)	F- Ratio
Treatment	Tr. SS	$(mn - 1)$	$MTr.SS = \frac{Tr.SS}{(mn - 1)}$	$\frac{MTr.SS}{M\text{Er. SS}}$
Row Blocks	RBSS	$(m - 1)$	$MRBSS = \frac{RBSS}{(m - 1)}$	$\frac{MRBSS}{M\text{Er. SS}}$
Column Blocks	CBSS	$(n - 1)$	$MCBSS = \frac{CBSS}{(n - 1)}$	$\frac{MCBSS}{M\text{Er. SS}}$
Rows	RSS	$(mn - 1)$	$MRSS = \frac{RSS}{(mn - 1)}$	$\frac{MRSS}{M\text{Er. SS}}$
Columns	CSS	$(mn - 1)$	$MCSS = \frac{CSS}{(mn - 1)}$	$\frac{MCSS}{M\text{Er. SS}}$
Subzone	SZSS	$(mn - 1)$	$MSZSS = \frac{SZSS}{(mn - 1)}$	$\frac{MSZSS}{M\text{Er. SS}}$
Error	Er. SS	$(mn - 2)^2 - (m+n-1)$	$M\text{Er. SS} = \frac{\text{Er. SS}}{(mn - 2)^2 - (m+n-1)}$	-
<b>Total</b>	<b>TSS</b>	<b><math>(mn)^2 - 1</math></b>	<b>-</b>	<b>-</b>

### 3.2 Sudoku Design with Type-2 Model

$$Y_{ij(klpr)} = \mu + \alpha_l + \beta_p + \tau_k + r(\alpha)_{l(i)} + c(\beta)_{p(j)} + s_q + e_{ij(klprq)}, \dots \quad (3.2)$$

where,

$\mu$  = General mean effect

$\alpha_l$  =  $l^{\text{th}}$  Row block effect

 $s_q$  =  $q^{\text{th}}$  Sub zone effect

 $\beta_p$  =  $p^{\text{th}}$  Column block effect

 $e_{ij}$  = Error with mean zero and variance  $\sigma^2$ 
 $\tau_k$  =  $k^{\text{th}}$  Treatment effect

 $r(\alpha)_{i(l)}$  =  $i^{\text{th}}$  row effect nested in  $l^{\text{th}}$  block (row)

From the above Sudoku square design model of order  $m \times n$ , it is assumed that the row effects are nested in the row block effects and the column effects are nested in the column block effect.

 $c(\beta)_{j(p)}$  =  $j^{\text{th}}$  column effect nested in  $p^{\text{th}}$  block (column)

After a little algebra we have obtained the formula for various sum of squares and degree of freedom

$$\text{Tr. SS} = \sum_{k=1}^{mn} \frac{Tr_k^2}{mn} - CF; \quad Tr_k^2 = \text{Sum square of } k^{\text{th}} \text{ treatment total, } k = 1, 2, \dots, mn$$

$$\text{RBSS} = \sum_{l=1}^m \frac{RB_l^2}{m n^2} - CF; \quad RB_l^2 = \text{sum square of } l^{\text{th}} \text{ Row block, } l = 1, 2, \dots, m$$

$$\text{CBSS} = \sum_{p=1}^n \frac{CB_p^2}{n m^2} - CF; \quad CB_p^2 = \text{sum square of } p^{\text{th}} \text{ Column block, } p = 1, 2, \dots, n$$

$$\text{RSS}_n = \sum_{l=1}^m \sum_{i=1}^n \frac{Ri_{i(l)}^2}{mn} - \sum_{l=1}^m \frac{RB_l^2}{m n^2}$$

$$\text{CSS}_n = \sum_{p=1}^n \sum_{j=1}^m \frac{Cj_{j(p)}^2}{mn} - \sum_{p=1}^n \frac{CB_p^2}{n m^2}$$

$$\text{SZSS} = \sum_{q=1}^{mn} \frac{S_q^2}{mn} - CF; \quad S_q^2 = \text{Sum square of } q^{\text{th}} \text{ Sub zone total, } q = 1, 2, \dots, mn$$

$$\text{Er. SS} = \text{TSS} - (\text{Tr. SS} + \text{RBSS} + \text{CBSS} + \text{RSS}_n + \text{CSS}_n + \text{SZSS})$$

**Table 2. ANOVA table for Sudoku design with Type - 2 model**

Source of variance	Sum of Squares (SS)	Degree of freedom (df.)	Mean sum of square (MSS)	F- Ratio
Treatment	Tr. SS	$(mn - 1)$	$MTr. SS = \frac{Tr. SS}{(mn - 1)}$	$\frac{MTr. SS}{MEr. SS}$
Row Blocks	RBSS	$(m - 1)$	$MRBSS = \frac{RBSS}{(m - 1)}$	$\frac{MRBSS}{MEr. SS}$
Column Blocks	CBSS	$(n - 1)$	$MCBSS = \frac{CBSS}{(n - 1)}$	$\frac{MCBSS}{MEr. SS}$
Rows within BR	$\text{RSS}_n$	$n(m-1)$	$\text{MRSS}_n = \frac{\text{RSS}_n}{n(m-1)}$	$\frac{\text{MRSS}_n}{MEr. SS}$
Column within BC	$\text{CSS}_n$	$m(n-1)$	$\text{MCSS}_n = \frac{\text{CSS}_n}{m(n-1)}$	$\frac{\text{MCSS}_n}{MEr. SS}$
Subzone	SZSS	$(mn - 1)$	$\text{MSZSS} = \frac{\text{SZSS}}{(mn - 1)}$	$\frac{\text{MSZSS}}{MEr. SS}$
Error	Er. SS	$(mn - 2)^2 - 1$	$\text{MEr. SS} = \frac{\text{Er. SS}}{(mn-2)^2 - 1}$	-
<b>Total</b>	<b>TSS</b>	<b><math>(mn)^2 - 1</math></b>	<b>-</b>	<b>-</b>

### 3.3 Sudoku Design with Type-3 Model

$$Y_{ij(klpr)} = \mu + \alpha_l + \beta_p + \tau_k + r_i + c_j + S(\alpha)_{l(q)} + D(\beta)_{j(r)} + e_{ij(klpq)} \quad \dots \quad (3.3)$$

where,

$\mu$  = General mean effect

$\alpha_l$  =  $l^{\text{th}}$  Row block effect

$\beta_p$  =  $p^{\text{th}}$  Column block effect

$\tau_k$  =  $k^{\text{th}}$  Treatment effect

$r_i$  =  $i^{\text{th}}$  Row effect

$c_j$  =  $j^{\text{th}}$  Column effect

$s(\alpha)_{l(q)}$  =  $l^{\text{th}}$  horizontal square effect nested in  $q^{\text{th}}$  row block effect

$D(\beta)_{p(r)}$  =  $p^{\text{th}}$  vertical square effect nested in  $r^{\text{th}}$  column block effect

$e_{ij}$  = Error with mean zero and variance  $\sigma^2$

From the above Sudoku square design model of order  $m \times n$ , it is assumed that the horizontal effects are nested in the row block effects, and the vertical are nested in the column block effect.

After a little algebra, we have obtained the formula for various sums of squares and degree of freedom

$$\text{Tr. SS} = \sum_{k=1}^{mn} \frac{Tr_k^2}{mn} - CF; \quad Tr_k^2 = \text{Sum square of } k^{\text{th}} \text{ treatment total, } k = 1, 2, \dots, mn$$

$$\text{RBSS} = \sum_{l=1}^m \frac{RB_l^2}{m n^2} - CF; \quad RB_l^2 = \text{sum square of } l^{\text{th}} \text{ Row block, } l = 1, 2, \dots, m$$

$$\text{CBSS} = \sum_{p=1}^n \frac{CB_p^2}{n m^2} - CF; \quad CB_p^2 = \text{sum square of } p^{\text{th}} \text{ Column block, } p = 1, 2, \dots, n$$

$$\text{RSS} = \sum_{i=1}^{mn} \frac{R_i^2}{mn} - C F; \quad R_i^2 = \text{Sum square of } i^{\text{th}} \text{ row total, } i = 1, 2, \dots, mn$$

$$\text{CSS} = \sum_{j=1}^{mn} \frac{C_j^2}{mn} - CF; \quad C_j^2 = \text{Sum square of } j^{\text{th}} \text{ column total, } j = 1, 2, \dots, mn$$

$$\text{HZSS}_n = \sum_{l=1}^m \sum_{q=1}^n \frac{S_{q(l)}^2}{mn} - \sum_{l=1}^m \frac{RB_l^2}{m n^2}$$

$$\text{VZSS}_n = \sum_{p=1}^n \sum_{r=1}^m \frac{D_{r(p)}^2}{mn} - \sum_{p=1}^n \frac{CB_p^2}{n m^2}$$

$$\text{Er. SS} = \text{TSS} - (\text{Tr. SS} + \text{RBSS} + \text{CBSS} + \text{RSS} + \text{CSS} + \text{HZSS}_n + \text{VZSS}_n)$$

**Table 3. ANOVA table for Sudoku design with Type - 3 model**

Source of variance	Sum of Squares (SS)	Degree of freedom (df.)	Mean sum of square (MSS)	F- Ratio
Treatment	Tr. SS	$(mn - 1)$	$MTr. SS = \frac{Tr. SS}{(mn - 1)}$	$\frac{MTr. SS}{MEr. SS}$
Row Blocks	RBSS	$(m - 1)$	$MRBSS = \frac{RBSS}{(m - 1)}$	$\frac{MRBSS}{MEr. SS}$
Column Blocks	CBSS	$(n - 1)$	$MCBSS = \frac{CBSS}{(n - 1)}$	$\frac{MCBSS}{MEr. SS}$
Rows	RSS	$(mn - 1)$	$MRSS = \frac{RSS}{(mn - 1)}$	$\frac{MRSS}{MEr. SS}$
Columns	CSS	$(mn - 1)$	$MCSS = \frac{CSS}{(mn - 1)}$	$\frac{MCSS}{MEr. SS}$
H-Square within RB	$\text{HZSS}_n$	$m(n-1)$	$\text{MHZSS}_n = \frac{\text{HZSS}_n}{m(n-1)}$	$\frac{\text{MHZSS}_n}{MEr. SS}$
V- Square within CB	$\text{VZSS}_n$	$n(m-1)$	$\text{MVZSS}_n = \frac{\text{VZSS}_n}{n(m-1)}$	$\frac{\text{MVZSS}_n}{MEr. SS}$
Error	Er. SS	$(mn - 2)^2 - mn$	$MEr. SS = \frac{Er. SS}{(mn-2)^2 - mn}$	-
Total	TSS	$(mn)^2 - 1$	-	-

### 3.4 Sudoku Design with Type- 4 Model

$$Y_{ij(klpr)} = \mu + \alpha_l + \beta_p + \tau_k + r(\alpha)_{l(i)} + c(\beta)_{p(j)} + S(\alpha)_{l(q)} + D(\beta)_{j(r)} + e_{ij(klpq)} \dots \quad (3.4)$$

Where,  $\mu$  = General mean effect

$\alpha_l$  =  $l^{\text{th}}$  Row block effect

$\beta_p$  =  $p^{\text{th}}$  Column block effect

$\tau_k$  =  $k^{\text{th}}$  Treatment effect

$r(\alpha)_{l(i)}$  =  $i^{\text{th}}$  row effect nested in  $l^{\text{th}}$  block (row)

$c(\beta)_{j(p)}$  =  $j^{\text{th}}$  column effect nested in  $p^{\text{th}}$  block (column)

$s(\alpha)_{l(q)}$  =  $l^{\text{th}}$  horizontal square effect nested in  $q^{\text{th}}$  row block effect

$D(\beta)_{p(r)}$  =  $p^{\text{th}}$  vertical square effect nested in  $r^{\text{th}}$  column block effect

$e_{ij}$  = Error with mean zero and variance  $\sigma^2$

In the above model, it is assumed that the row and horizontal square effects are nested within the row block effects. In contrast, the column effects and vertical square effects are nested within the column block effects. The Sudoku design of order  $m \times n$ . After a bit of algebra, we have obtained the formula for various sum of squares and degree of freedom.

$$\text{Tr. SS} = \sum_{k=1}^{mn} \frac{Tr_k^2}{mn} - CF; \quad Tr_k^2 = \text{Sum square of } k^{\text{th}} \text{ treatment total, } k = 1, 2, \dots, mn$$

$$\text{RBSS} = \sum_{l=1}^m \frac{RB_l^2}{m n^2} - CF; \quad RB_l^2 = \text{sum square of } l^{\text{th}} \text{ Row block, } l = 1, 2, \dots, m$$

$$\text{CBSS} = \sum_{p=1}^n \frac{CB_p^2}{n m^2} - CF; \quad CB_p^2 = \text{sum square of } p^{\text{th}} \text{ Column block, } p = 1, 2, \dots, n$$

$$\text{RSS}_n = \sum_{l=1}^m \sum_{i=1}^n \frac{Ri_{l(i)}^2}{mn} - \sum_{l=1}^m \frac{RB_l^2}{m n^2}$$

$$\text{CSS}_n = \sum_{l=1}^m \sum_{j=1}^n \frac{Cj_{j(p)}^2}{mn} - \sum_{p=1}^n \frac{CB_p^2}{n m^2}$$

$$\text{HZSS}_n = \sum_{l=1}^m \sum_{q=1}^n \frac{Sq_{l(l)}^2}{mn} - \sum_{l=1}^m \frac{RB_l^2}{m n^2}$$

$$\text{VZSS}_n = \sum_{p=1}^n \sum_{r=1}^m \frac{Dr_{r(p)}^2}{mn} - \sum_{p=1}^n \frac{CB_p^2}{n m^2}$$

$$\text{Er. SS} = \text{TSS} - (\text{Tr. SS} + \text{RBSS} + \text{CBSS} + \text{RSS}_n + \text{CSS}_n + \text{HZSS}_n + \text{VZSS}_n)$$

**Table 4. ANOVA table for Sudoku design with Type - 4 model**

Source of variance	Sum of Squares (SS)	Degree of freedom (df.)	Mean sum of square (MSS)	F- Ratio
Treatment	Tr. SS	$(mn - 1)$	$MTr. SS = \frac{\text{Tr. SS}}{(mn - 1)}$	$\frac{MTr. SS}{MEr. SS}$
Row Blocks	RBSS	$(m - 1)$	$MRBSS = \frac{\text{RBSS}}{(m - 1)}$	$\frac{MRBSS}{MEr. SS}$
Column Blocks	CBSS	$(n - 1)$	$MCBSS = \frac{\text{CBSS}}{(n - 1)}$	$\frac{MCBSS}{MEr. SS}$
Rows within BR	$\text{RSS}_n$	$n(m-1)$	$\text{MRSS}_n = \frac{\text{RSS}_n}{n(m-1)}$	$\frac{\text{MRSS}_n}{MEr. SS}$
Column within BC	$\text{CSS}_n$	$m(n-1)$	$\text{MCSS}_n = \frac{\text{CSS}_n}{m(n-1)}$	$\frac{\text{MCSS}_n}{MEr. SS}$
H-Square within RB	$\text{HZSS}_n$	$m(n-1)$	$\text{MHZSS}_n = \frac{\text{HZSS}_n}{m(n-1)}$	$\frac{\text{HZSS}_n}{MEr. SS}$
V- Square within CB	$\text{VZSS}_n$	$n(m-1)$	$\text{MVZSS}_n = \frac{\text{VZSS}_n}{n(m-1)}$	$\frac{\text{VZSS}_n}{MEr. SS}$
Error	Er. SS	$(mn - 2)^2 - mn + m + n - 2$	$MEr. SS = \frac{\text{Er. SS}}{(mn-2)^2 - mn + m + n - 2}$	-
Total	TSS	$(mn)^2 - 1$	-	-

## 4. RESULTS AND DISCUSSION

The analysis of experimental data presented in below layout has been analyzed by four different ANOVA models, as mentioned in the material and method section.

### 4.1 Type-1 Model as Given in Chapter 3

Grand total (GT) = 1418.

Total sum of square (TSS) = 3925.26

Treatment sum of square (Tr. SS) = 3340.50

Row sum of square (RSS) = 106.32

Column sum of square (CSS) = 73.51

Sum of square due to row block (RBSS) = 88.36

Sum of square due to column block (CBSS) = 4.98

Sum of squares due to sub zones (SZSS) = 196.27

Error sum of square (Er. SS) = 115.29

The table illustrates that the effects of treatment mean values, rows, and sub-zones differ significantly at the 5 % level of significance. The table shows the F calculated values for the treatment groups, which are 69.53, for the rows, which are 4.59, and for the sub-zones, which are 4.08, respectively. The table also shows that

there are six sources of variations present other than the error. The sources like Row Block, Column Block and Sub-zones are addition to the traditional row column designs (e.g., LSD). Among these additional sources, the only the effect of the zone Shows a significant different at the 5% level. The treatment effect also shows highly significant difference among themselves. In Type 1 model, the effects of three additional sources viz., row block, column block and sub zones, can be successfully estimated. However, among the additional effects, only sub-zones are significantly different.

### 4.2 Type-2 Model as Given in Chapter 3

Since those calculations have been done previously, the same exact details are not written out repeatedly in every analysis to avoid repetition.

Sum of square due to row block (RBSS) = 88.36

Sum of square due to column block (CBSS) = 4.98

Sum of square due to row effect nested in row block ( $RSS_n$ ) = 17.96

Sum of square due to column effect nested in column block ( $CSS_n$ ) = 68.52

Error sum of square (Er. SS) = 208.64

Yield of paddy (q. ha <sup>-1</sup> )					
39.5(6)	30.8(1)	36.2(2)	40.6(3)	57.9(4)	49.2(5)
54.6(3)	42(4)	52.3(5)	37.4(6)	44.3(1)	27.9(2)
37(1)	35(2)	47(3)	41(4)	43.5(5)	42.2(6)
49(4)	54(5)	34(6)	32(1)	30.9(2)	48(3)
52.3(5)	43.3(6)	26(1)	28(2)	44(3)	49.5(4)
31.5(2)	46(3)	51.5(4)	51.6(5)	30.5(6)	33.2(1)

**Fig. 2. Layout of the experiment with yield of paddy (q. ha<sup>-1</sup>)**

*The figures in parentheses indicate the treatment numbers*

**Table 5. ANOVA table for Sudoku design with Type- 1 model**

Source of variance	Sum of Squares (SS)	Degree of freedom (df.)	Mean sum of squares (MSS)	F- Ratio
Treatment (Tr. SS)	3340.50	5	668.10	69.53*
Row Blocks	88.36	2	44.18	2.21
Column Blocks	4.98	1	4.98	1.53
Rows	106.32	5	21.26	4.59*
Columns	73.51	5	14.70	0.51
Subzone	196.27	5	39.25	4.08*
Error	115.29	12	9.60	-
<b>Total</b>	<b>3925.26</b>	<b>35</b>	<b>-</b>	<b>-</b>

*The asterisks (\*) indicate significance at 5% level*



**Table 6. ANOVA table for Sudoku design with Type- 2 model**

Source of variance	Sum of Squares (SS)	Degree of freedom (df.)	Mean sum of squares (MSS)	F- Ratio
Treatment	3340.50	5	668.10	48.03*
Row Blocks	88.36	2	44.18	3.17*
Column Blocks	4.98	1	4.98	0.35
Rows within BR	17.96	3	5.98	0.43
Column within BC	68.52	4	17.13	1.23
Subzone	196.27	5	39.25	2.82
Error	208.64	15	13.90	-
<b>Total</b>	<b>3925.26</b>	<b>35</b>	<b>-</b>	<b>-</b>

The asterisks (\*) indicate significance at 5% level.

The table illustrates that effects of treatment mean values and row block differ significantly at the 5 % level of significance. The table shows the F calculated values for the treatment groups, which are 48.03, and for the row block, which is 3.17, respectively. The table also shows that there are six sources of variations present other than the error. The sources like Row Block, Column Block, Rows within BR, Columns within BC, and Sub-zone are additions to the traditional row-column designs (e.g., LSD). Among these additional sources, only the row block shows a significant difference at the 5 % level. The treatment effect also shows highly significant differences among themselves. In the Type 2 model, five additional sources are considered and also estimated successfully, viz., Row block, column block, Rows within block row (BR), Column within block column (BC), and Sub zones. Among the additional effects, only row blocks are significantly different.

### 4.3 Type-3 Model as given in Chapter 3

The sum of square due to Horizontal effect nested in row block ( $HZSS_n$ ) = 107.91

The sum of square due to vertical effect nested in column block ( $VZSS_n$ ) = 191.28

Error sum of square (Er. SS) = 80.89

The table illustrates that the effects of treatment mean values, row blocks, H- square within RB, and V- square within CB differ significantly at the 5 % significance level. The table shows the F calculated values for the treatment groups is 82.59, for the row block, is 5.46, for the H- square within RB is 4.44, and for the V- square within CB is 5.91, respectively. The table also shows that there are seven sources of variations present other than the error. The sources like Row Block, Column Block, H- square within RB, and V- square within CB are addition to the traditional row-column designs (e.g., LSD). Among these additional sources, row block, H- square within RB, and V- square within CB show significant differences at a 5% level. The treatment effect also shows highly significant differences among themselves. Type 3 model evaluates four additional sources of variation viz., Row block, Column block, Horizontal Square within row block (H- Square within RB), and Vertical Square within column block (V- Square within CB). Among them, Row block, H-Square, within RB, and V- Square within CB are significantly different.

### 4.4 Type-4 Model as Given in Chapter 3

Error sum of square (Er. SS) = 105.71

**Table 7. ANOVA table for Sudoku design with Type- 3 model**

Source of variance	Sum of Squares (SS)	Degree of freedom (df.)	Mean sum of squares (MSS)	F- Ratio
Treatment	3340.50	5	668.10	82.59*
Row Blocks	88.36	2	44.18	5.46*
Column Blocks	4.98	1	4.98	0.61
Rows	106.32	5	21.26	2.62
Columns	73.51	5	14.70	1.81
H-Square within RB	107.91	3	35.97	4.44*
V- Square within CB	191.28	4	47.82	5.91*
Error	80.88	10	8.88	-
<b>Total</b>	<b>3925.26</b>	<b>35</b>	<b>-</b>	<b>-</b>

The asterisks (\*) indicate significance at 5% level

**Table 8. ANOVA table for Sudoku design with Type - 4 model**

Source of variance	Sum of Squares (SS)	Degree of freedom (df.)	Mean sum of square (MSS)	F- Ratio
Treatment	3340.50	5	668.10	82.15*
Row Blocks	88.36	2	44.18	5.43*
Column Blocks	4.98	1	4.98	0.61
Rows within BR	17.96	3	5.98	0.73
Column within BC	68.52	4	17.13	2.10
H-Square within RB	107.91	3	35.97	4.42*
V- Square within CB	191.28	4	47.82	5.88*
Error	105.71	13	8.13	-
<b>Total</b>	<b>3925.26</b>	<b>35</b>	<b>-</b>	<b>-</b>

*The asterisks (\*) indicate significance at 5% level*

Table 8 illustrates that the effects of treatment mean values, row blocks, H- square within RB, and V- square within CB differ significantly at the 5 % significance level. The table shows the F calculated values for the treatment groups is 82.15, for the row block is 5.43, for the H- square within RB is 4.42, and for the V- square within CB is 5.88, respectively. The table also shows seven sources of variations present other than the error. The sources like Row Block, Column Block, rows within BR, columns within BC, H-square within RB, and V- square within CB are additions to the traditional row-column designs (e.g., LSD). Among these additional sources, row Block, H-square within RB, and V- square within CB show significant difference at a 5 % level. The treatment effect also shows highly significant differences among themselves. The most comprehensive model, type -4, assesses six additional sources of variation viz., Row block, Column block, Rows within BR, Column within BC, H-Square within RB, V-Square within, and CB can be successfully estimated. In the context of further effect, the additional effects, Row block, H-Square within RB, and V-Square within CB, are significantly different. An evaluation of field experiments through four types of Sudoku square design models with rectangular subzones reveals that these designs enable to estimate the effects of a large number of sources, which are not achievable through traditional row-column experimental designs such as LSD. These models are particularly effective for their ability to extract more nuanced information from experimental setup to extract more nuanced information from experimental setups with an equivalent compared to a row-column design (LSD). It is proven that the mean square error values for all four models are substantially lower

than those associated with an LSD design. The four models demonstrate that the additional sources of variation analyzed are not accounted for by a traditional row-column design such as the Latin square design. This design approach promises to improve the quality of experimental outcomes and offers new opportunities for more in-depth investigations. This capability underscores the potential of this design to provide more comprehensive insights, promising alternatives compared to traditional experimental row-column design, and also alternatives for complex experimental frameworks in agricultural research.

## 5. CONCLUSION

This research paper focuses on the construction and analysis of Sudoku designs with rectangular sub-zones and their application in agricultural experiments. The study is conducted in the New Alluvial Zone of West Bengal, particularly on paddy fields. The main objective is to explore the effectiveness of these designs in agricultural research and to examine their statistical properties. Sudoku designs are useful for experimental layouts because they help control variability in field trials. Traditional Sudoku squares are divided into equal sub-squares, but in this study, the sub-zones have rectangular shapes. The research explains how these modified Sudoku designs can be constructed and analyzed mathematically. The study describes various methods to generate Sudoku designs with rectangular sub-zones. It also evaluates their efficiency in reducing experimental errors. The properties of these designs are examined using statistical tools to determine their suitability

for agricultural experiments. The results show that Sudoku designs with rectangular sub-zones provide a systematic way to arrange treatments in field experiments. This arrangement helps in reducing variability and improving the accuracy of experimental outcomes. The study concludes that these designs are beneficial for agricultural research, particularly in optimizing paddy field experiments.

Overall, this research highlights the potential of using Sudoku-based experimental designs in agriculture. It suggests that these designs can improve the reliability of field experiments and contribute to better agricultural practices.

## DISCLAIMER (ARTIFICIAL INTELLIGENCE)

Author(s) hereby declares that NO generative AI technologies such as large Language Models (ChatGPT, COPILOT, etc.) and text-to-image generation have been used during writing or editing of this manuscript.

## COMPETING INTERESTS

Authors have declared that no competing interests exist.

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